Convexity Analysis in Detecting a Steel Plant Hidden Global Optimum

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Abstract. This case study demonstrates the value of classical analysis and to a lesser degree, system decomposition for finding a global optimum missed by a sequential linear programming scheme which converges to a non-global local minimum. The example is a 20 variable steelmaking problem in which the variable annual cost to be minimized is linear, as are all constraints except a non-convex one in each blast furnace. The sequential linear programming method gives a proven *local* minimum, although the non-convex nonlinearity prevents any proof of global optimality. The proven *global* minimum found here has a 4% lower cost. The local *min*imum costs only 0.2% per annum less than the rather flat global *max*imum, so the original local minimization only achieved about 5% of the economy possible. In the overall plant, the cost saving is over three million US$ (1972) annually.

Key words. Global optimization, convex analysis, convex-concave, blast furnace, steel plant.

Introduction

Good industrial optimization has two elements: an accurate engineering model and ways, both theoretical and computational, to optimize the model. Studies performed by engineers do not always give enough attention to the optimization step, especially as the model gets large and nonlinear. Busy developing the model, the engineers may neglect the mathematical analysis and place unwarranted reliance on the computer code. Such codes, inherited from the operations research community, perform so well on the convex problems for which they were designed that it is tempting to misapply them to nonconvex problems for which they were never intended. This, of course, risks convergence to a local optimum which is not globally optimal, a danger than has recently encouraged research on algorithms with global optimization properties (Horst & Tuy, 1990, henceforth abbreviated H&T).

The present article, echoing a sentiment expressed earlier in the context of engineering design (Wilde, 1978), advises engineers (and operations analysts) to invest a little time in preliminary mathematical analysis before turning to the computer. This is not so much to save computation costs, although on large problems this may be justified, as to make sure that the algorithms are used correctly.

The theory required is not as difficult as that already mastered by most engineers during their education, and to the professional mathematician it borders on the trivial. What is needed is not sophisticated analysis, but the motivation to look for and exploit such simple properties as monotonicity and convexity (or concavity). Thus monotonicity analysis (Papalambros & Wilde, 1988, henceforth abbreviated P&W) may find, without computation, many of the constraints that must be active at a global minimum, greatly reducing the complexity that can lead to computational error. A similar study of convexity and concavity, which for reference will be known here as *convexity analysis* will often determine limits on the use of computer codes requiring convexity.

This approach is illustrated by a case study of a 20 variable iron and steel plant taken from a chemical engineering optimization text (Ray & Szekeley, 1973, henceforth abbreviated R&S). In the present article, convexity analysis finds the globally minimal production schedule to be quite different from, and much better than, the plan generated in the original study. The problem with the earlier study was that it uncritically applied a gradient projection code to a model which, although mostly linear, has concave regions which trapped the convex algorithm at a non-optimal local minimum.

This article is intended, however, to be more than a cautionary tale to encourage careful preliminary analysis before turning to the computer. It is also meant to provide a detailed model for testing newer global optimization algorithms requiring neither convexity nor preliminary analysis. This model would be a vehicle for testing answers to the research problem which broadly stated is, "Should algorithms automate preliminary analysis or seek to avoid it entirely?". A good trial for a candidate algorithm would be to start it at the wrong local minimum to see if, how and when it finds the global minimum. Especially impressive would be any algorithm succeeding on the full 20 variable model in its original form with no simplification or decomposition.

Another goal of this article is to record the transformations needed to expose the convex-concave character of the troublesome nonlinearity. Thermodynamically motivated, these *intrinsic* rational transformations have general interest for global optimization theory because in this study at least they simplify the analysis by separating the variables. The second derivative analysis of the convex-concave function is also included, despite its elementary character, because this classical method was not mentioned in the H&T survey of d.c. (difference of convex) functions.

The Problem

This case study is a 20 variable steelmaking problem involving two smelters (blast furnaces) feeding iron to two parallel steel furnaces: open hearth and basic oxygen. To be determined for a given steel production rate is the distribution of ore feed between sintered and pelleted types, together with production rates for all four units and feed rates for coke, silicon carbide and steel scrap, as shown in Figure 1. The variable annual cost to be minimized is linear, as are all constraints except a non-convex one for each blast furnace.

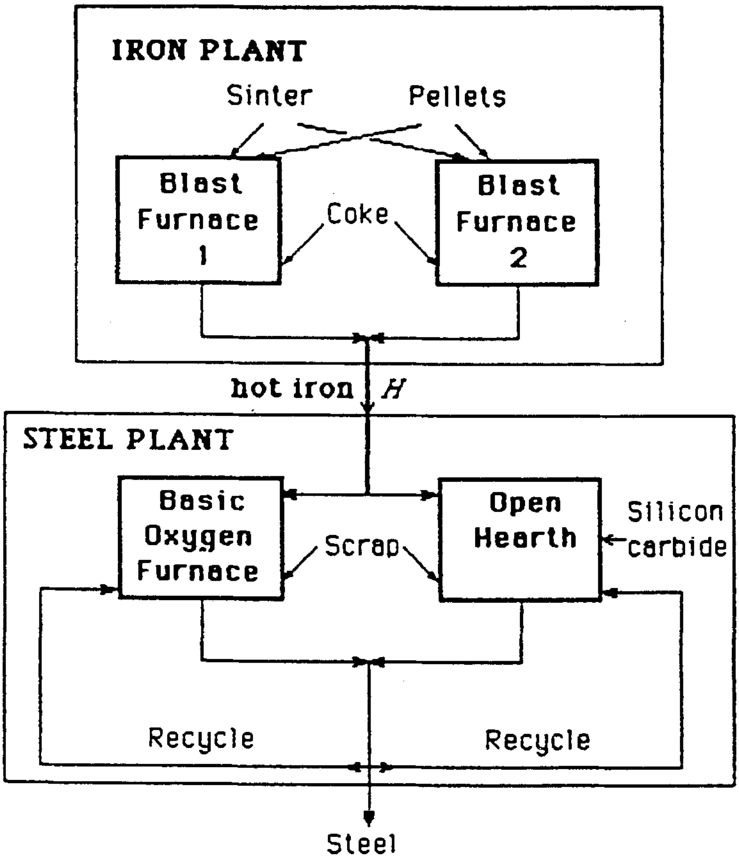

Fig. 1. Iron and steel process flow diagram.

Along with the definitions of the variables, all equations will be listed in the sections following, which are organized according to the order in which the analysis proceeds. Symbols and variables are listed in the Notation and Nomenclature section. To avoid needless numerical complexity and to permit direct comparison of this article with R&S, we follow their practice of expressing mass in English tons (908 kg) or M tons (10^6 English tons) instead of SI units.

Branch Decomposition by Iron Rate Parametrization

First the convergent branch structure (Wilde & Beightler, 1967, pp. 399–406) where the blast furnaces combine hot iron to feed the steel plants is exploited. Fixing the unknown hot iron rates X_4 and X_8 would decouple the nonlinear blast furnaces from the linear steel plants. This permits each blast furnace to be analyzed separately for each value of its hot iron rate, now treated as a coupling parameter and consequently symbolized as a capital letter as suggested by P&W

(p. 6). This distinguishes the iron rates from the system variables whose values are free to be optimized. Then the total hot iron rate

$$H = X_4 + X_8 \tag{1}$$

becomes a parameter in the linear program describing the rest of the system, the steel plant.

Solving this system numerically would in principle require solving, for every feasible combination of X_4 and X_8, a nonlinear optimization problem for each blast furnace and a linear program for the remaining steel plant. Preliminary convexity analysis will, however, generate the need for only one single-dimensional root-finding in each blast furnace.

The Blast Furnace Models

The mathematical models for the two blast furnaces (henceforth abbreviated BF1 and BF2) differ not in form but only in a few parameter values. The BFs smelt iron from sintered (iron) ore, pelleted ore and coke (carbon), whose annual feed rates are respectively represented by x_1, x_2 and x_3 for BF1 and x_5, x_6 and x_7 for BF2. The variable production costs v_1 and v_2 reflecting the cost of raw materials are

$$v_1 = 21x_1 + 30x_2 + 25x_3 , \tag{2-1}$$

$$v_2 = 21x_5 + 30x_6 + 25x_7 . \tag{2-2}$$

R&S assume a BF fixed cost, not needed in this analysis since it is constant, but no other production cost.

The mass balances have the same form for both BFs.

$$0.715x_1 + 0.91x_2 = X_4 , \tag{3-1}$$

$$0.715x_5 + 0.91x_6 = X_8 . \tag{3-2}$$

However, upper bounds on the sinter feed rates differ

$$x_1 \leqslant 1.7 , \tag{4-1}$$

$$x_5 \leqslant 0.8 , \tag{4-2}$$

as do capacity constraints on the iron rates

$$0.7 \leqslant X_4 \leqslant 1.6 , \tag{5-1}$$

$$0.4 \leq X_8 \leq 0.8 . \tag{5-2}$$

All the preceding relations are linear; the only nonlinearities involve the ratios of coke feed to hot iron rates. In BF1

$$x_3/X_4 = 0.4[1 + 0.5\exp(-0.7x_2/x_1)] + 0.3(1 - X_4)^2 , \tag{6-1}$$

whereas in BF2 the relation is

$$x_7/X_8 = 0.5[1 + 0.4\exp(-0.7x_6/x_5)] + 0.35(0.5 - X_8)^2 . \tag{6-2}$$

Concerning Equations (6-1), R&S say on p. 302, "An increase in the fraction of pellets tends to reduce the coke rate to a limiting value of 0.4. . . . The second term on the right-hand side indicates that for a fixed pellet-sinter ratio the coke rate shows a minimum at a particular production rate – 1 M tons/annum. . . ."

Intrinsic Transformation

Since relations (6) both have the same form, only BF1 will be analyzed in detail. BF1, in addition to a coupling parameter X_4, has three feed variables constrained by a linear mass balance (3-1) and a coke rate Equation (6-1), leaving one degree of freedom for optimization of the linear cost. Elimination x_2 and x_3 gives a nonlinear cost depending only on the variable x_1 and the parameter X_4.

$$v_1 = 43X_4 + 7.5\, X_4(1 - X_4)^2 - 2.57x_1 + 8.73X_4 \exp(-0.78X_4/x_1) . \tag{7}$$

This expression is clarified by a nonlinear change of variable commonly employed by chemical engineers. For each BF define the ratio

$$r_1 = x_1/X_4 , \tag{8-1}$$

$$r_2 = x_5/X_8 . \tag{8-2}$$

Then division of (7) throughout by X_4 gives

$$v_1/X_4 = 43 + 7.5(1 - X_4)^2 - 2.57r_1 + 8.73\exp(-0.78/r_1) , \tag{9}$$

which can be optimized with respect to the new system variable r_1 independently of the iron production rate X_4. Abbreviate

$$u_1(r_1) = -2.57r_1 + 8.73\exp(-0.78/r_1) \tag{10}$$

so that

$$v_1 = X_4[43 + 7.5(1 - X_4)^2 + u_1(r_1)]\,. \tag{11}$$

Since X_4 is positive, BF1 cost v_1 is clearly minimum with respect to r_1 when $u_1(r_1)$ is also minimum with respect to r_1.

The three terms in brackets on the right of Equation (11) have the dimensions of cost per unit mass of iron produced. Thus the ore and coke costs are $43/ton when no sinter is used, $7.5(1 - X_4)^2$ is the additional cost depending on BF flow, and $u_1(r_1)$ represents the unit cost variation which depends on ore composition.

Equality constraints (3) evaluated for x_2, $x_6 = 0$ gives the upper bound

$$r_1, r_2 < (0.715)^{-1} = 1.40\,. \tag{12}$$

Since sintered ore does not have to be used, r_1 and r_2 can take any non-negative values satisfying (12).

Transforming the problem from one involving mass flow rates to one involving dimensionless composition ratios is a mathematical manifestation of a thermodynamic principle. This is that chemical processes depend on *intrinsic* properties such as composition rather than *extrinsic* properties proportional to amounts present, flow rates in this example. For this physical reason the transformation used here is called "intrinsic", although its use of a ratio could well lead to its characterization by mathematicians as a "rational" transformation. The intrinsic transformation is recommended here to optimization modelers as a possible way to separate variables, particularly parameters from system variables.

Convexity Analysis

The composition cost function $u_1(r_1)$ will now be analyzed for convexity by studying its first two derivatives in this range. They are

$$\mathrm{d}u_1/\mathrm{d}r = -2.57 + 6.81 r_1^{-2} \exp(-0.78/r_1) \tag{13}$$

and

$$\mathrm{d}^2u_1/\mathrm{d}r^2 = 6.81(0.78 - 2r_1)r_1^{-4} \exp(-0.78/r_1)\,. \tag{14}$$

The sign of $\mathrm{d}^2u_1/\mathrm{d}r^2$ is the same as that of the linear factor $0.78 - 2r_1$. Thus $u_1(r_1)$ is convex-concave; convex at the low end of its domain and concave elsewhere, with an inflection at $r_1 = 0.39$.

Over the domain $0.39 \leq r_1 \leq 1.42$ in which r_1 is concave, the global minimum must be at one of the two extremes (H&T, p. 10). Direct evaluation gives $u_1(0.39) = 0.135 \leq u_1(1.42) = 0.573$, indicating an artificially constrained regional minimum at the inflection 0.39. The first derivative there certainly cannot be negative or even zero, for this and the negativity of d^2u_1/dr^2 would allow u_1 to decrease locally as r_1 increases. Hence $du_1/dr_1 > 0$ at the inflection, ruling it out as a candidate for global minimality in the region of convexity $0 \leqslant r_1 \leqslant 0.39$. Therefore the global minimum r_1^{**} must be in the restricted domain $0 \leqslant r_1^{**} \leqslant 0.39$ where u_1 is known to be convex. By any number of numerical methods the global minimum u_1^{**} can be found to be at the unique stationary point $r_1^{**} = 0.18$

$$u_1^{**} = u_1(r_1^{**}) = u_1(0.18) = -0.348 . \tag{15-1}$$

Equations (8-1), (3-1) and (6-1) give the globally optimal BF1 sinter, pellet and coke rates as functions of the as yet undetermined parameter X_4:

$$x_1 = 0.18X_4 ; \quad x_2 = 0.96X_4 ; \quad x_3 = [0.408 + 0.3(1 - X_4)^2]X_4 . \tag{16-1}$$

A similar analysis of BF2 gives a unique inflection at $r_2 = 0.385$ and the global minimum

$$u_2^{**} = u_2(r_2^{**}) = u_2(0.157) = -0.343 . \tag{15-2}$$

As functions of the parameter X_8 the corresponding feed components are

$$x_5 = 0.157X_8 ; \quad x_6 = 0.98X_8 ; \quad x_7 = [0.503 + 0.35(0.5 - X_8)^2]X_8 . \tag{16-2}$$

Figure 2 graphs both $u_1(r_1)$ and $u_2(r_2)$. It is particularly interesting that for

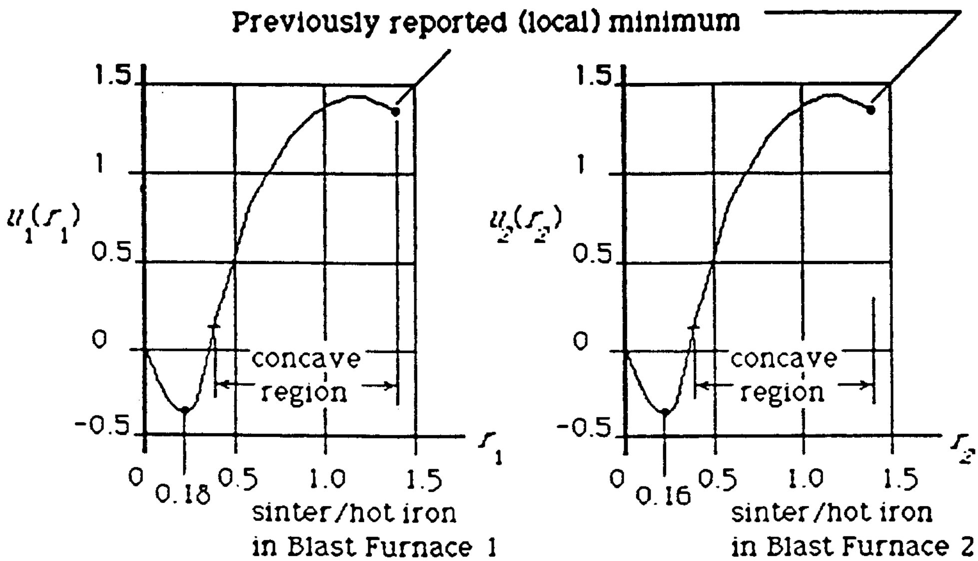

Fig. 2. Blast furnace composition costs.

both BFs the algorithm of Ray & Szekeley, not being restricted to the convex region, converged to the nonglobal local minimum at the upper end of the domain of concavity, the very region ruled out by the convexity analysis. The cost saving is about 4% in BF1. Worthy of note is that the local minimum costs only 0.2% less than the rather flat global *maximum*. Thus the original local minimization only achieved about 5% of the economy possible. The situation in the second blast furnace is similar. This costly oversight is a powerful argument for preliminary convexity analysis, even at the elementary mathematical level used here.

Monotonicity and Convexity Analysis of the Hot Iron Rates

Substitution of the optimized composition costs u_1^{**} and u_2^{**} into the BF cost functions of Equations (2-1), (3-1) and (6-1) gives expressions cubic in parametric hot iron rates

$$v_1 = 50.15X_4 - 15.0X_4^2 + 7.5X_4^3 , \tag{17-1}$$

$$v_2 = 47.4X_8 - 8.75X_8^2 + 8.75X_8^3 . \tag{17-2}$$

To prove monotonicity, shift the origins to the minimum production rates given by constraints (5-1):

$$Y_1 = X_8 - 0.7 \qquad Y_2 = H_2 - 0.4 .$$

The unit cost first derivatives are then

$$\mathrm{d}v_1/\mathrm{d}Y_1 = 40.18 + 1.50Y_1 + 22.5Y_1^2 > 0 , \tag{18-1}$$

$$\mathrm{d}v_2/\mathrm{d}Y_2 = 46.61 + 3.50Y_2 + 26.25Y_2^2 > 0 . \tag{18-2}$$

Their positivities prove that the unit costs strictly increase with production rate. The rates must therefore satisfy the mass balance with strict equality:

$$Y_1 + Y_2 = H - 1.1 \equiv Y \geqslant 0 . \tag{19}$$

Moreover, the first derivatives are also strictly increasing, so the separability of $v_b = v_1 + v_2$ implies its strict convexity.

The actual optimization of the total cost subject to the constraints will be performed after the optimizing value of H has been determined. Meanwhile it is important to notice that Equation (18-1) implies that hot iron cannot be produced any cheaper than $40.18/ton, the lowest possible cost rate where $X_1 = 0$. That is,

$$\mathrm{d}(v_b^{**})/\mathrm{d}H > 40.2 . \tag{20}$$

This fact will be needed later to determine H. It remains to determine the best value of the parametric hot iron rate H taking the steel plant into account.

Sub-Optimization of the Steel Plant

With the total iron rate treated as a parameter, the steel plant becomes completely linear. Hence by linear programming it is possible to compute the minimum cost schedule for the steel plant for any numerical value of H. Added to the corresponding optimal hot iron cost $v^{**}(H)$, this optimal steel plant cost gives the best total cost for the H given, so selecting the value H^{**} minimizing this sum determines the global minimum cost schedule for the combined iron and steel plants. The availability of good linear programming codes makes this the fastest practical way to a numerical solution, since the preliminary analysis guarantees that the result obtained will be globally optimal.

There is, however, a surprisingly powerful method available for solving this problem analytically, thereby gaining deeper insight into the problem structure. The sparsity of the steel plant equations allows using *monotonicity analysis* (Papalambros & Wilde, 1988) to find the complete pattern of constraint activity in the two steel furnaces without performing any linear programming computations. The linearity of all these constraints permits their solution in closed form, giving all variables and the steel plant cost as linear functions of the parametric hot iron flow from the blast furnaces. Since the total cost increases monotonically with iron rate, the smallest feasible rate is globally optimal. The optimizing value of this parameter then completely determines the globally minimal cost production schedule. Although the results of the monotonicity analysis will be given in this article, the details will be omitted to save space.

Steel Plant Model

R&S formulate the steel plant with the twelve variables x_9 through x_{20} listed in the Nomenclature. They then derive and list the model constraints and describe it qualitatively in physical and economic terms as an engineer. Here equality constraints will be labelled "E", inequalities, "I".

First consider the *parametric* constraints, so called because they involve the parameters external to the steel plant, here the finished steel demand (3.0) given as a constant and the hot metal rate M given as a coupling parameter to be determined later by overall system optimization. The finishing plant yield of 70% gives, for the product rate 3.0 specified, one strict equality determining the crude steel rates

$$x_{13} + x_{17} = 3.0/0.70 = 4.286 \tag{E1}$$

and another giving the recycled home scrap production.

$$x_{10} + x_{15} = 0.3(4.286) = 1.286\,. \tag{E2}$$

A third mass balance couples the steel plant to BFs.

$$x_9 + x_{16} = H\,. \tag{E3}$$

Next consider the non-parametric or *local* constraints. In the basic oxygen furnace (BOF) there are

a mass balance $X_{13} = 0.90(x_9 + X_{10} + X_{11})\,,$ (E4)

a hot iron limit $x_9 \geqslant 4(x_{10} + x_{11} - 12x_{12})\,,$ (I1)

a production limit $x_{13} \leqslant 3.5\,,$ (I2)

and an SiC limit $x_{12} \leqslant x_9/24\,.$ (I3)

In R&S Equation (8.4.16) the coefficient $(24)^{-1}$ is misprinted as 24.

In the open hearth (OH) furnace the only explicit constraints are a mass balance,

$$x_{17} = 0.92(x_{14} + x_{15} + x_{16}) \tag{E5}$$

and a capacity limit

$$2.0 \geqslant x_{16} + 1.33(x_{14} + x_{15})\,. \tag{I4}$$

The steel plant variable cost to be minimized reflects the high cost of the SiC additive and relative economy of the more modern BOF

$$v_s \equiv 180x_{12} + 15x_{13} + 26x_{17}\,, \tag{0}$$

Steel Plant Results

To save space, the results of the steel plant analysis are presented here without the intermediate details. These results could be confirmed numerically by solving linear programs for a range of values of the coupling parameter, the hot iron rate H. The active constraints, aside from the parametric constraints E1, E2 and E3 known in advance to be active, turn out to be the remaining BOF and OH mass balances E4 and E5, together with the inequalities I1 (BOF hot iron max), I3 (SiC max), I4 (OH capacity) and the non-negativity of the OH hot iron. Using these linear equations to eliminate all but the BOF home scrap rate x_{10} and the parametric hot iron rate H reduces the steel plant cost to

$$v_s = 118.6 - 13.1H\,, \tag{21}$$

subject to

$$1.87 \leqslant H \leqslant 2.24\,, \tag{22}$$

$$x_{10} \leqslant 1.286\,, \tag{23}$$

Thus the solution is not unique; any value in the range $0 \leqslant x_{10} \leqslant 1.286$ is optimal.

Table I. Production schedules

Unit Variable	Present study		R&S		Remarks
	Rate	Cost	Rate	Cost	
BF1 Sinter	0.23	4.8	1.70	35.7	Upper bound = 1.7
BF1 Pellet	1.23	36.8	0	0	
BF1 Coke	0.55	13.8	0.804	20.1	
BF1 Total cost		55.4		55.8	
BF1 Hot iron	1.28	(43.3)	1.215	(45.9)	2.6 saving
BF2 Sinter	0.093	2.0	0.564	11.8	
BF2 Pellet	0.58	17.4	0	0	
BF2 Coke	0.30	7.5	0.274	6.8	
BF2 Total cost		26.9		18.6	
BF2 Hot iron	0.59	(45.4)	0.403	(46.2)	0.8 saving
Total hot iron	1.87	82.3(44.0)	1.618	74.4(46.0)	2.0 saving
BOF Hot iron	1.87	82.3	1.618	74.4	
BOF SiC	0.078	14.0	0.1	18.0	
BOF Iron/SiC	24.0		16.18		R&S infeasible (<24)
			0.25	11.6	Hot iron shortage
			[−0.022]	[−4.0]	SiC adjustment
BOF Home scrap	0	0	0	0	
BOF Bought scrap	1.40	0	[1.605]	0	Original plan
			[−0.22]	0	Scrap adjustment
			1.38	0	Adjusted plan
BOF Crude steel	2.93	44.0	2.902	43.5	Upper bound = 3.5
Total BOF cost		140.3(47.9)	143.5	(49.4)	Includes iron cost
OH Hot iron	0	0	0	0	
OH Home scrap	1.286	0	1.285	0	
OH Bought scrap	0.219	0	0.218	0	
OH Crude steel	1.39	36.0	1.383	36.0	
OH Capacity	2.0		2.0		At upper bound
Total OH cost	36.0	($26)	36.0	($26)	Operating cost only
Total Crude steel	4.29	176.3	4.29	179.5	$3.2(10^6)/yr saving
Home scrap	1.29		1.29		30% recycled
Finished steel	3.00	(58.8)	3.00	(59.8)	1.0 reduction

Rates in 10^6 tons/year. Costs in 10^6 US dollars (c. 1971). Numbers in parentheses ( ) are in $/ton.

Optimizing the Coupling Parameter Value

Equation (21) shows that the minimum steel plant variable cost v_s *de*creases with H, whereas Equation (20) proves that the total blast furnace cost *in*creases with H. Adding the two costs gives the system cost v to be minimized with respect to H

$$v = v_s + v_b = 118.6 - 13.1H + v_b ,$$

Differentiation of this total with respect to H shows that despite the negative coefficient of H in the steel plant cost, inequality (20) implies that the total cost strictly increases with H

$$\mathrm{d}v/\mathrm{d}H > 40.2 - 13.1 = 27.1 > 0 .$$

Hence H must be made as small as possible. By inequality (22) it follows that the globally minimizing value is $H^{**} = 1.87$. This value contradicts the 1.618 found by R&S (p. 312), which gives an infeasible schedule violating the SiC constraint (I3). The misprint noted there may have been coded into their computation.

The corresponding steel production schedule is generated by working through the eliminated active equations. Together with the BF schedule to be generated next, this steel production schedule is displayed in Table I. The steel plant component of the minimum variable cost is $v_s = 118.6 - 13.1(1.87) = 94.1$ million dollars annually.

Blast Furnace Rate Optimization

The optimal BF rates for $H^{**} = 1.87$ are obtained by minimizing the convex BF total variable cost $v_1(H_1) + v_2(H_2)$ from Equations (17-1) subject to the mass balance $H_1 + H_2 = 1.87$. The results are $H_1^{**} = 1.28$ and $H_2^{**} = 0.59$, from which Equations (16-1) generate the BF feed schedule displayed in the "Present Study" column of Table I. BF total variable cost from Equations (17-1) is $v = 55.4 + 26.9 = 82.3$ million dollars annually. Adding the 94.1 from the optimized steel plant gives a total variable cost of 176.3 million\$/year.

Comparison

The column labelled "R&S" in Table I contains the production schedule obtained by Ray and Szekeley to allow comparison with the present study. This is a bit artificial because the R&S steel plant schedule uses more SiC than permitted by the SiC limit (I3). This falsely allows some of the expensive hot iron to be replaced by scrap which is free, giving a schedule which, although apparently cheaper, would not really meet the steel demand.

To permit easy although not quite accurate comparison, the correct steel plant schedule of the present study was combined with the R&S blast furnace schedule to obtain a feasible plan. The SiC rate was lowered as needed, and the additional hot iron required was charged as it it were purchased externally at the cost of that produced by R&S. With this simplication, favorable to the corrected R&S plan, one can say that the globally minimum cost schedule costs US$3.2 million less per annum, at least, than that obtained by the linearization algorithm. This would reduce the unit variable steel cost by at least one US dollar per ton.

An environmental bonus would be the reduced coke consumption of the optimal schedule (21.3 tons/yr) compared to the 26.9 tons/yr needed by R&S even to produce their infeasibly small amount of iron. The extra iron needed by R&S to meet the steel demand would of course require even more coke somewhere else. The wasted carbon not only costs money; it loads the atmosphere with carbon dioxide, thereby contributing unnecessarily to the greenhouse effect.

Concluding Discussion

This study illuminates the interplay between analysis and computation in global optimization. Convex problems can be solved entirely by computer because there local minimization algorithms find the global optimum. But when convexity is missing it is dangerous to apply convex algorithms to approximations that are convex only locally. Instead one must proceed cautiously with the most careful analysis possible under the circumstances before resorting to the computer.

In the present study the structure permitted decomposition of the problem into three subproblems: the two nonlinear blast furnaces and the linear steel plant. This decoupling did more than save computation; it was essential for isolating the nonlinearities for correct analysis. The nonlinear analysis itself required using the constraints to reduce the number of variables from four to two. Then functional analysis permitted concentration of all the nonlinearity into a partial optimization with respect to a single variable, for which the analysis could have been entirely graphical. The nonlinear analysis needed was elementary, requiring only routine study of a second partial derivative. Of theoretical interest to global optimizers is that the function was dc, a difference of convex functions, although this property was not used in its optimization. The resulting subproblem of finding the blast furnace outputs was shown to have an increasing convex objective and a single linear constraint, parametric in a single coupling variable. The linear steel plant can be solved either by monotonicity analysis or computed by a parametric sequence of linear programs.

The model is presented to the global optimization community as a test problem for general global optimization algorithms that rely on computation rather than analysis. The challenge would be to find the global optimum computationally using the original formulation with its score of variables and pair of nonlinear constraints.

It may be only rarely that a large complicated problem can be reduced to one in a single variable as was possible here. But reduction is worth trying, especially on the largely monotonic but non-convex problems encountered in industry, for then at least one can prove that the solution obtained is optimum globally. The three million dollar per year saving obtained here is, of course, a powerful economic incentive for careful analysis.

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Notation and Nomenclature

Mass rates are in (10^6) tons/year; costs in \$/ton; \$ represent US\$ c. 1970. Parameters are capitalized.

Symbol	*Name of variable*
H	combined hot iron rate $(X_4 + X_8)$
r_1, r_2	sinter/iron ratio for BF1 and BF2
u_1, u_2	unit composition cost variation for BF1 and BF2
v_1, v_2	variable feed cost for BF1 and BF2
$v_b(H)$	total variable feed cost for both blast furnaces
$v_s(H)$	total variable cost for both steel furnaces
x_1	sintered iron ore rate into BF1
x_2	pelleted iron ore rate into BF1
x_3	coke rate into BF1
X_4	hot iron rate from BF1
x_5	sintered iron ore rate into BF2
x_6	pelleted iron ore rate into BF2
x_7	coke rate into BF2
X_8	hot iron rate from BF2
x_9	hot iron to basic oxygen furnace (BOF)
x_{10}	home scrap to BOF
x_{11}	bought scrap to BOF
x_{12}	silicon carbide to BOF
x_{13}	crude steel from BOF
x_{14}	home scrap to open-hearth furnace (OH)
x_{15}	bought scrap to OH
x_{16}	hot iron to OH
x_{17}	crude steel from OH
x_{18}	total crude steel
x_{19}	total home scrap
x_{20}	total bought scrap
Y_1, Y_2	additional hot iron from BF1 and BF2

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